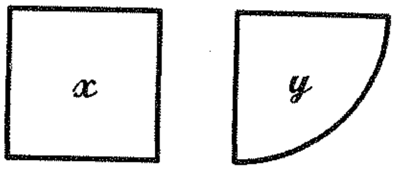
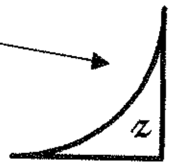
Sheet D1 A New Unit of Area

So far we have used these basic shapes to measure the areas of other figures.



Now we introduce a new basic shape: the **corner**. It is left when a quadrant is cut out of a square. We'll call its area z.



For example, we have already found that tile A has area

$a = x + 2y$ (a 'square' and two 'quadrants')

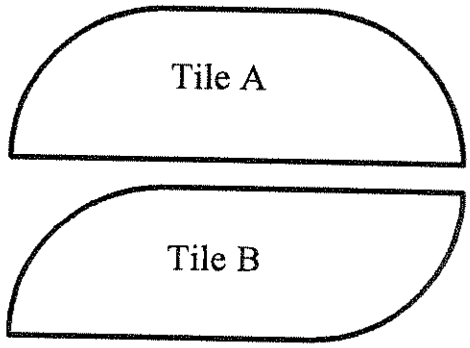
Now we can see that its area is also given by

$a = 3x - 2z$ (3 'squares', less two 'corners'.)

and $a = 3y + z$ (three 'quadrants' and one 'corner').

Similarly, $b = 3x - 2z$

and $b = 3y + z$



Look at the tiles with areas c, d and e:

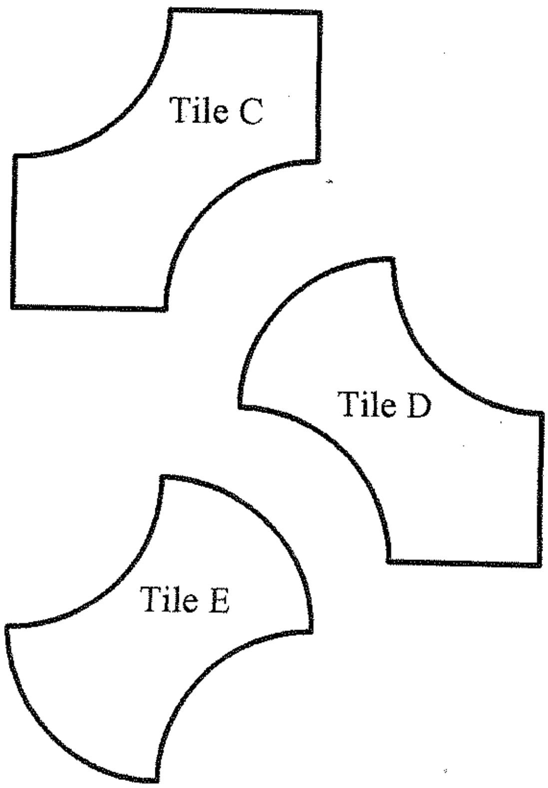
Show how these can be split into three basic shapes.

Fill in an expression below which gives the area of each one without using subtraction.

$c =$

$d =$

$e =$



Remember:

Always *simplify* your answer as much as possible.

<u>For example</u>: $x + y + z + z$

can be simplified to $x + y + 2z$.

And this equals $2x + z$ (because $y + z = x$).

Sheet D2 Easy Substitution

The area of each tile can be expressed in terms of the areas of <u>any pair</u> of the three **basic shapes.**

For example, we have seen that the area of Tile A can be given in terms of:

	x and y	by	$(x + 2y)$	a 'square' and two 'quadrants'
<u>or</u>	x and z	by	$(3x - 2z)$	3 'squares' with two 'corners' taken off
<u>or</u>	y and z	by	$(3y + z)$	three 'quadrants' and one 'corner'.

As these three expressions all refer to the **same** area, $x + 2y = 3x - 2z = 3y + z$.

Express the areas of tiles B, C, D and E (**after simplification**) in terms of

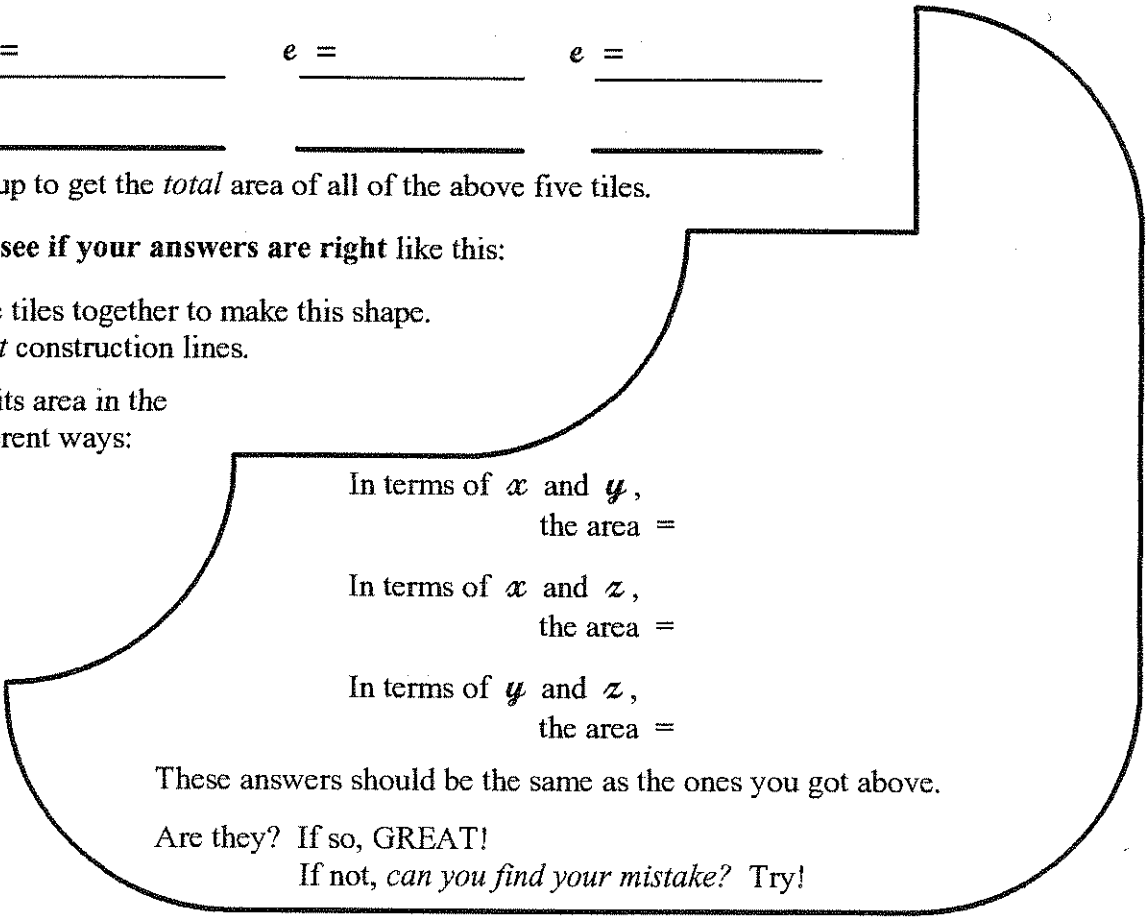
(i) x and y:	(ii) x and z:	(iii) y and z:
$a = x + 2y$	$a = 3x - 2z$	$a = 3y + z$
$b =$	$b =$	$b =$
$c =$	$c =$	$c =$
$d =$	$d =$	$d =$
$e =$ ______________	$e =$ ______________	$e =$ ______________
______________	______________	______________

Now add up to get the *total* area of all of the above five tiles.

Check to see if your answers are right like this:

Fit all five tiles together to make this shape.
Draw *light* construction lines.

Count up its area in the three different ways:



In terms of x and y,
the area =

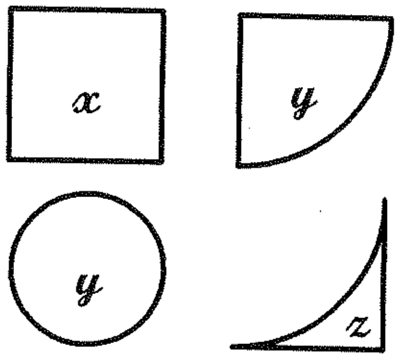
In terms of x and z,
the area =

In terms of y and z,
the area =

These answers should be the same as the ones you got above.

Are they? If so, GREAT!
If not, *can you find your mistake?* Try!

Sheet D3 More Difficult Substitution

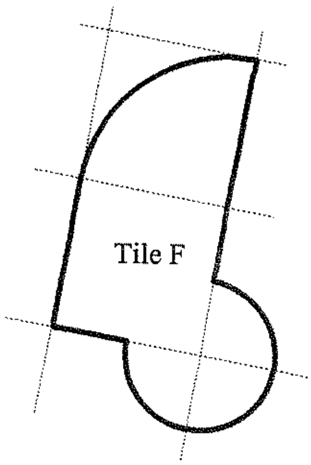


Working out the areas of the last three tiles is more difficult.

Start by drawing the grid lightly in pencil. Then you can see more easily the parts that make up the complete tile.

Worked example:

Consider Tile F. The grid is already drawn in.



Its area f = the area of a quadrant + the area of a square + three quarters of the area of a small circle

In other words $f = y + x + \frac{3}{4}y$ which we can ***simplify*** to: $f = x + 1\frac{3}{4}y$

To express this area in terms of x and z, we remember that

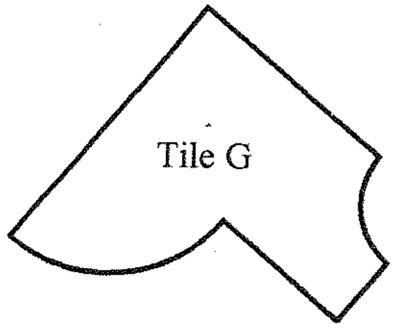
$$y = x - z.$$

So for y we ***substitute*** $(x - z)$ in the above expression.

This gives us: $f = x + 1\frac{3}{4}(x - z)$

Simplifying the right hand side, we have: $x + 1\frac{3}{4}x - 1\frac{3}{4}z$,

so $f = 2\frac{3}{4}x - 1\frac{3}{4}z$.

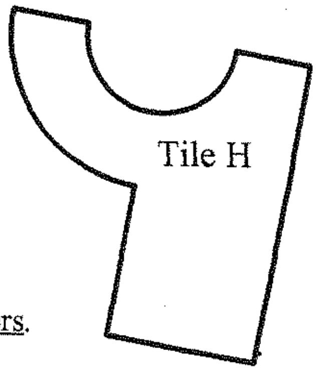


Find the areas of tiles G and H in terms of x and z.

$g =$ $h =$

$=$ $=$

$=$ $=$



Now work out their areas in terms of y and z.

$g =$ $h =$

$=$ $=$

$=$ $=$

REMEMBER Letters like x, y, and z always stand for actual <u>numbers</u>.

Sheet D4 Checking Your Answer

By means of substitution and simplification, the areas of each of the other three tiles can be expressed in terms of the *three basic shapes* in exactly the same way.

Remember the **substitution formulae**:

$$x = y + z \qquad z = x - y \qquad y = x - z$$

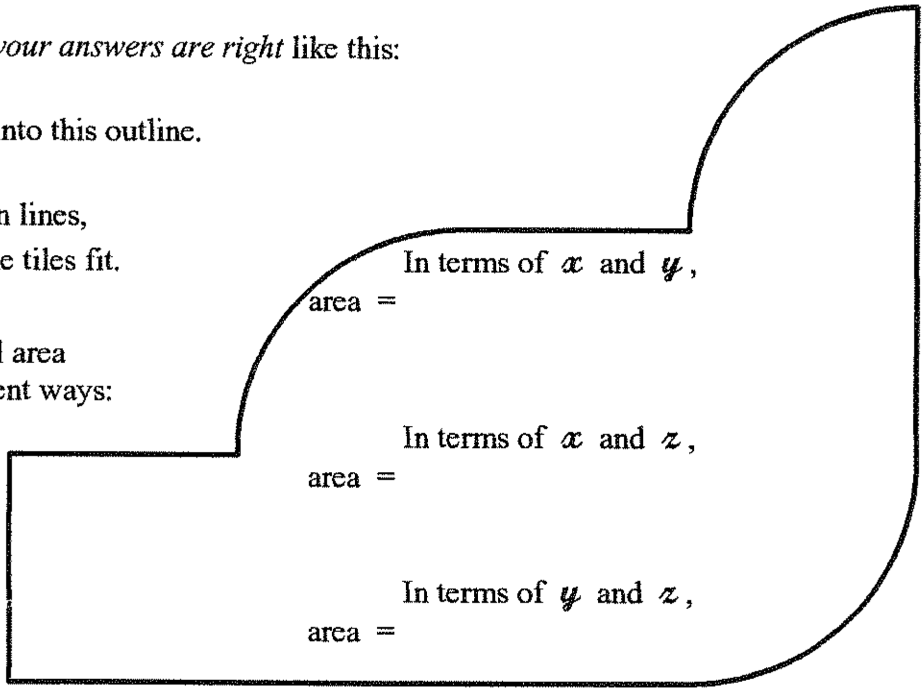
which you can use to eliminate any one of the three letters x, y and z.

Try this out for yourself. Express the areas of tiles F, G and H (after simplification) in terms of

(i) x and y,	(ii) x and z,	(iii) y and z.
$f =$	$f =$	$f =$
$g =$	$g =$	$g =$
$h =$	$h =$	$h =$
Totals: ______	______	______

Now *check to see if your answers are right* like this:

1. Fit all three tiles into this outline.
2. Draw construction lines, and show how the tiles fit.
3. Count up the total area in the three different ways:



These answers should be the same as the ones you obtained in the first bit of work above.

Are they? If so, *GREAT!*
If not, *can you find your mistake?* Try!

Sheet D5 Working with Shapes and Areas

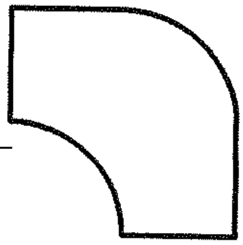
Draw *construction lines* to help you do these examples.

1. Find the area of this shape.

2. $x + y + z$ Draw an interesting shape with this area.

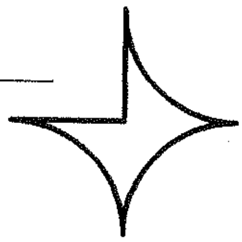
Now go on and do these in the same way:

3.



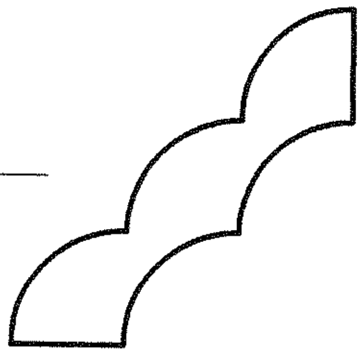
4. $x + y - z$

5.



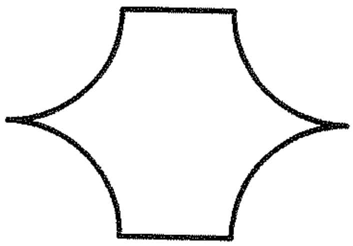
6. $2x - y + 3z$

7.



8. $2y + 2z$

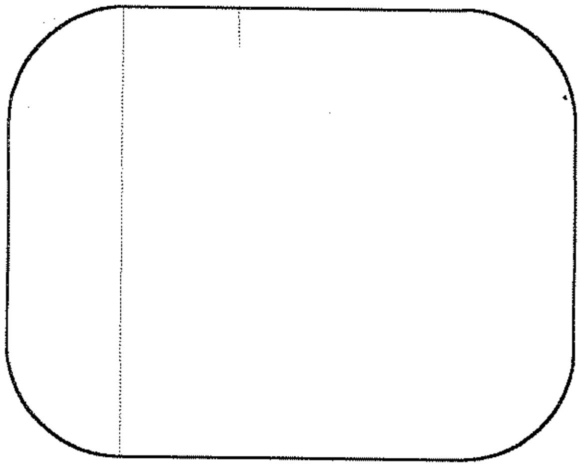
9.



10. $y + 2z$

Sheet D6 The Original TakTiles

Originally the eight TakTiles were cut from a panel of this shape.

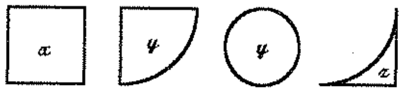


By drawing *light* construction lines to guide you, (one has already been drawn in) work out the <u>total area</u> of this panel, and write it below in terms of:

(i) x and y, (ii) x and z, (iii) y and z.

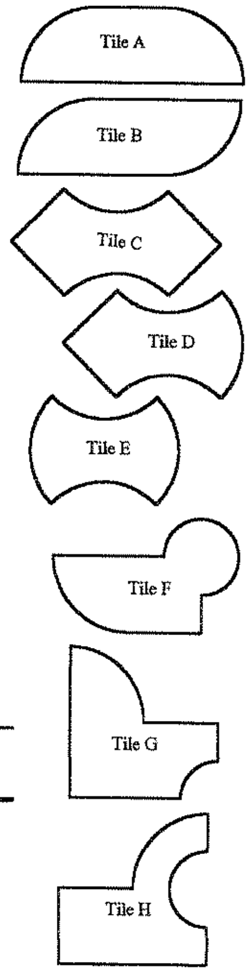
Area = = =

Now put the tiles together to make this 'rounded rectangle', and show in the diagram how they fit.



Write down the area of each tile in terms of :

(i) x and y, (ii) x and z, (iii) y and z.



(i)	(ii)	(iii)
a =	a =	a =
b =	b =	b =
c =	c =	c =
d =	d =	d =
e =	e =	e =
f =	f =	f =
g =	g =	g =
h =	h =	h =

Now add up each column.

Are your three totals the same as above? They should be!

ALGEBRA THROUGH GEOMETRY

NAME(S): ....**CLASS:** ....

Each square is the same size as the shape named ***x***.
The sheet can help you work out the area of each shape.
You might trace the shapes and then remove them to work out the area.